THE NEW
MATHEMATICS
OF ARCHITECTURE
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with 628 illustrations, 435 in colour
It has long been said that architecture is a game played with clear objectives, but no guiding set of rules. Mathematics, on the other hand, has forever been described by its believers as a form of knowledge best understood as a game with lots of rules, but no clear objective. For evidence of the enduring beauty of this paradoxical combination of two distinct (interwoven, even if opposing) human endeavours, look no further than this wonderful book, which Jane and Mark Burry have edited as an invaluable contribution to mathematics in the architecture of our time.

Early on in the 20th century, Le Corbusier observed that, however different or new modern architecture was, mathematics was still at its heart. It seemed an obvious point for the Swiss master to make then, and one no more (or less) surprising today, when we remember that architects have needed mathematics since at least the time when one of their kind drew a right angle with a stick in the sand and realized numbers went a long way to help reliably communicate an idea to someone else (and recall, this is what architects most fundamentally do: they communicate ideas — instructions — to others). That mathematics is an important part of architecture is pretty obvious a point to a field of enquiry whose own origins lie with the publication of Vitruvius’ Ten Books some two thousand years ago, made up as it is by a series of (numerically ordered) universal formulae and mathematical explanations explaining how architects might best arrange architectural matter in meaningful, lasting ways. In other words, to say that mathematics is integral to architecture is like saying numbers are helpful when trying to count.

So what’s the big deal about architecture and mathematics conversing in the ways they do today? Plenty, I’d say, without one having to judge a book by (the title on) its cover. The most surprising feature of a recent and evolutionary leap in Le Corbusier’s ‘weapons of the gods’ is the simultaneity of, on the one hand, a growing power and complexity of mathematical processes in architecture, and, on the other, a marked disappearance of numbers themselves within its own language and discourses. Architects no longer speak the language, as did their ancestors, of whole numbers or predictable geometries, any more than they do of drawings made at fixed dimension or measurable scales. This situation owes itself to another kind of contemporary revolution of course: architects’ near-universal assimilation of digital, information-based design platforms in their studios, which have now become the basis for not only architectural practice, but, as well, the very communication media through which their ideas are now conceived, flow and proliferate.

In considering the ubiquity of computing and information environments that keep this information behind what a user sees on a software interface) has, accordingly, left a very real void in the language of architecture. And this is where the projects that follow offer such a valuable contribution: not only in the accomplishments of their form, beauty or material realization, but also in their willingness to take on directly the challenges of re-inventing the very language — and numbers — lying hidden behind their surface.
We have reached the end of a decade and a half in which digital computation has given architects new creative opportunities with which to access the geometrical space opened up by post-17th-century mathematicians. The resulting new wave of interest in the relationship of mathematics to space-making has been aesthetically driven, and yet its expression has transcended the metaphorical. It has found expression from within the process of making as a new species of architecture, and has infiltrated architectural processes in ways that have forged radical change. This book is an account of the ways in which this new mathematical focus has manifested in designed and built projects since the mid-1990s. Had it been written ten years earlier, the phenomenon would have been fresh and urgent, but the book itself would have its deepest roots embedded in geometry. Architecture has always been very closely related; both disciplines, while closely related, developed in response to different relationships among things. Mathematics has been concerned with the creation of space; mathematics with its geometry has been practiced since prehistoric times (in building, for example), but that the Greeks took it further, abstracting and inscribing sacred meaning in significant buildings. And what of architecture? In the human mind between seven fish in a river and seven days, a landmark advance was achieved: the idea of a week. Arithmetic is also from the Greek, derived from the word ἀριθμός meaning ‘land measurement’ or ‘earth measure’. The Greek historian Herodotus (485–425 bc) attributed the origin of geometry to the Egyptians’ apportionment of land by equal measure and the relationship of the revenue claimed by the pharaoh according to the land area and any reduction in that area, duly measured by the pharaoh’s overseers, during the annual flooding of the Nile. So geometry is considered to be descended from the concrete subdivision and organization of space. In this sense, architecture and geometry are mutually implicated in their conception and development. Both have the power to express and organize space using concepts outside the constraints of a direct mapping to a physical representation. The principal distinction lies in their levels of abstraction and generality. Geometry looks for generalities and, once established (demonstrated or proved), offers them up for use; architecture employs these general relationships constructively to underpin and create specific spatial relationships. It is assumed that practical geometry has been practised since prehistoric times (in building, for example), but that the Greeks took it further, abstracting and systematizing it and leaving us with Euclid’s axioms. Geometrical knowledge developed and crystallized.

‘Arithmetic’ is also from the Greek, derived from the world ἀριθμός meaning number. It is clear that the concept of number is deep in the human psyche; as architectural theorist Sanford Kwinter points out, ‘numbers give us dimensions and proportions, fix the geometry as shape, and inscribe sacred meaning in significant buildings. And what of algebra, the third constituent of mathematics?’ Ingeborg M. Rocker, in her article ‘When Code Matters’, has this to say: ‘Today, when architects calculate and exercise their thoughts, everything turns into algorithmic computation, the writing and reading of code through simple rules, plays an ever-increasing role in architecture.’

Algebra is the primary meta-language of mathematics, in which all geometrical objects and numbers are further abstracted and generalized. The formal (logical) languages of computer code are not algebra, but algebra has provided the language in which to couch all the spatial, proximal and numerical relationships the
Outside this critical framework, the imagination of many leading architectural figures was caught by the chaotic systems of Edward Norton Lorenz and the fractal geometries of Benoît Mandelbrot, whose essay ‘Fractional Form, Chance and Dimension’ (1975) provided the main conduit from complexity science to architecture. The three destabilizing ideas of discontinuity, recursivity and self-similarity were subsequently taken up by a string of architects as ideas for organizing principles. But by the 1990s, there was already a trend to deny the inspiration of chaos, and to dismiss a hash in seen as having little tangible connection to the central concerns of architecture. It was, in effect, an idea before its time. Given very few years and the computational means with which to explore the generative potential of recursive systems, these ideas re-entered architecture as if by stealth at a much more engaged level, and became part of the working design lexicon. Meanwhile, the distributed, networked and overlapping space of the post-30th-century human experience has brought us directly into the foreground when considering spatial models, and given architects, as space-makers, an entree into the conceptual spaces first defined by mathematicians and philosophers in the 19th century. Topology in architecture is no longer a critique of the power of the plane and gravitational vector in mainstream Modernism. Instead, it is the reality that we experience. Metrics and vectors have given place to distributed networks. This is a fundamentally different space in which to live. What is the significance for architecture? Philosophically, Gilles Deleuze gave us the ‘bodily’; any corporeal arrangement composed of an infinite number of parts that are held together when they move in unison at the same speed, more or less powerful, more or less able to affect change in their environment, depending on the degree to which they are capable of being affected themselves. Michael Speaks argues that contemporary architectural praxis as such a body becomes more powerful to the degree that it transforms the clutter of little truths into design intelligence. ‘Intelligence’ has become a word much overused in our time. It remains ill-defined, and in its current popular usage smacks of ‘artificial intelligence’, with the deliberate elision of the unpopular idea of artificial. The rise of the biological sciences has created a new atmosphere of respect for the living; new artificial systems are rated according to the extent to which they ape biological systems, a reversal of the situation that once pertained, whereby logic-based systems were looked to for insight into thought.

Dynamic, variable, spatial models are not new in spatial design, and are certainly not confined to electronic computation. But the power to integrate many variables, make links outside the confines of three geometrical dimensions, and simulate scenarios has given the mathematics–design nexus in its newly pluralist and agile manifestation is ubiquitous in this mission. This book is about architecture, but not exclusively about architects. In many of the projects on the following pages, the design teamwork between different professions is explicit. There is a historical perspective here. Robin Evans, in his book The Projective Cast, gives a fine account of the geometrical baton passing from the architects to the engineers. The two building-design disciplines were hitherto divided along civil and military lines, but after the 18th century, and specifically the work of the French engineer Gaspard Monge, the knowledge of descriptive geometry that had been so vital but subsequently lost in the architecture profession was appropriated by engineers, reappearing in the great shipbuilding and railway projects of the 19th century. Mathematical knowledge in construction design from this time onwards became more or less the exclusive domain of the engineers.
During the 20th century, built works outside the rectilinear
dogma of Modernism are attributable in great number to notable
engagement with mathematics, including Félix Candela, Pier Luigi Nervi,
Heinz Isler and Frei Otto, to name a few. What is interesting about
these engineers is their use of physical analogue models to find
structurally efficient shapes that provokes the general principle of
the model as a responsive dynamic system. Moreover the in the
recent history there have been architect-led works that espoused these
themes discussed in the chapters that follow, which were designed
without the power of digital computation — seminal works that
whisper of closer geometrical affinity to natural form. Why is Le
Corbusier’s chapel of Notre-Dame du Haut at Ronchamp absent from
the discosom of surfaces? Even more pressing, how can we discuss
dataease in the absence of the progenitor of all
datascapes – the Philips Pavilion for Expo 58 – a parametrically
created through inversion in the sphere, toroid surfaces used for
hyperbolic surfaces subtracted from the building mass, surfaces
for its complex configuration and symbolic mathematical identity,
defining and building free-form surfaces, a minimal surface used
in the architecture. They are diverse in the nature of that surface
shape of their curved surfaces is a principal expressive element
architecturally, rather than mathematically, selected, suggested by
the works themselves. What all the projects have in common in
the first chapter, Mathematical Surfaces and Seriality, is that the
subject of their curved surfaces is a principal expressive element
in the architecture. They are diverse in the nature of that surface
and its creation. All use mathematical rules or technique. Here
we see mathematics used to offer solutions to the ‘problem’ of
defining and building four-form surfaces, a minimal surface used
for its complex configuration and symbolic mathematical identity,
hyperbolic surfaces subtracted from the building mass, surfaces
created through inversion in the sphere, toroidal surfaces used for
their rational flexible qualities, and surfaces shaped by gravity to
solve the problem of structural minimalism.

The second chapter, Chaos, Complexity, Emergence, includes
projects in which fractals and self-similarity are at the heart of
their expression, and wherein the simple elements of a system
result in intriguing emergent results at another scale. Work that is
fundamentally digital in production is contrasted with that created
using more traditional techniques. ‘Packing and Tilting’ has some
overlap with the previous topic in its inclusion of fractal tilings, but
also introduces the phenomenon of periodicity–tilings and three-
dimensional space-filling packings that do not map to themselves
when translated. This contrasts with traditional Islamic tiling,
which, while it includes complex multiple symmetries, is generally
periodic. The exploration of periodicity is a relatively recent area of
mathematics that has been given architectural expression.

Optimization is perhaps the best example of the use of
computer in architecture. It includes projects in which divisive
optimization methods find structurally economical form, elegantly
resolve the parametrization of surfaces, minimize the size of structural
members, and resolve the perfect acoius space. In general, the
computer is used in a variety of ways to search a defined solution
space, and to sculpt material by responding to strain. ‘Topology’
is a collection of works that have been driven less by shape and
more by relationships of proximity and connection. Some projects
explicitly investigate the architectural possibilities of non-orientable
surfaces, such as the Möbius band and the Klein bottle – one
adopts knots and the other the multiple instances of a single,
topological description of a space. The final chapter, Datascapes
and Multi-dimensionality, is a rare collection of interactive spaces,
ranging from installations to urban design, which respond to stimuli
from approaching humans to atmospheric change. These are
genuinely multi-dimensional spaces by any mathematical definition
that test Theo van Doesburg’s ninth proposition in his manifestos,
‘Towards a Plactic Architecture’, published in De Stijl in 1924: ‘... with
the aid of calculation that is non-Euclidian and takes into
account the four dimensions, everything will be very easy.’

Reviewing the projects presented, there is a natural division
between those in which the primary mathematical constituent
is an idea, and those where mathematics is first and foremost
positioned as a problem-solver. In some, the two roles are balanced
or combined, and in all, the mathematical idea or problem-solver
is also instrumental in the design process and to the form of
the architectural outcome. In all this activity, there is no real
evidence of convergence between architect and mathematician,
but there is a sense in which mathematics and mathematical
designts have contributed to the formation and cohesion of diverse
creative teams. In 2000, Lionel March, a pioneer in the study of
mathematical applications in architecture using computation,
published an essay entitled Architecture and Mathematics Since
1960, a review of work in which March himself had been involved
and that encompassed a range of research from land use and
built-forms to more qualitative investigations of form based
on group theory of symmetries.15 He, child educationalist Friedrich
Froebel, who observed that relationals studies of form were ‘forms
of life’. In recent years, we have seen a steep uptake curve of
the application of mathematical thinking in architecture, as this
theoretically-grounded work is followed by a period in which
the mathematics has indeed come to life in the practice of architecture
at every creative level. We wait, breathless, to see how this will play
out in the working relationships and built environment in the years
to come.